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# Human Capital and Endogenous Growth in a Large-Scale Life-Cycle Model

Patricio Arrau

Life-cycle models of growth can yield a negative relation between population growth and income per capita growth, where the direction of causality goes from the exogenous rate of population growth to the endogenous rate of income growth. Tax policy can affect the proportion of human and physical capital in household portfolios. Tax policy that favors human capital over physical capital produces higher growth in per capita income.

Most models of economic growth are infinite horizon models that neglect the role of human capital in shaping life-cycle variables. Arrau introduces training decisions in a life-cycle model (the Auerbach-Kotlikoff simulation model) to study the role of human capital both in life-cycle behavior and as an engine of growth.

All the models of growth we have accumulated by studying aggregate models of growth could be greatly enhanced by studying models of growth at a more disaggregated level, he concludes.

The crucial assumption about growth of Arrau's model is that new generations are endowed with the average level of skills available when they were born.

He studies the impact of demographics and taxation on the endogenous rate of growth.

Population growth affects the age distribution of the population and the equilibrium spillover that sustains growth.

Unlike what happens with infinite horizon models, this model shows per capita income growth and population growth to be inversely related. Unlike what happens with recent fertility-based models, this model shows the direction of causality to go from exogenous population growth to endogenous economic growth.

To forgo consumption, households hold human and physical capital. Tax policy can affect the proportion of these assets in household portfolios. Tax policy that favors human capital (as opposed to physical capital) translates into higher per capita growth in income.

This paper is a product of the Debt and International Finance Division, International Economics Department. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Sheila King-Watson, room S8-025, extension 33730 (42 pages with figures and tables).

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## 1 Introduction

Human capital has played a dual role in the economic literature. On one hand it has been considered a fundamental source of aggregate growth<sup>1</sup>. On the other, it has been extensively used in the labor literature to explain the observed profile of earnings, work-time and training over the life-cycle<sup>2</sup>. In the context of a dynamic model, Blinder and Weiss (1976) and Ryder et. al. (1976) endogenize consumption, training and leisure decisions and study the profile of these variables over the life-cycle. A realistic profile is obtained where full-time training (schooling) occurs early in life, work time and (on-the-job) training occurs at middle age and a full retirement period is observed by the end of life.

The purpose of this paper is to build a model where the roles of human capital as an engine of growth and as a component of the life-cycle profile of earnings and labor supply are simultaneously treated. We aim to study in a general equilibrium setting the way the long-run rate of growth is related to structural, policy and demographic parameters. Following King and Rebelo (1988) we are interested in a model of long-run growth which satisfies Kaldor's stylized facts: constant interest rate, constant capital-income ratio and constant capital-labor ratio along the bal-

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<sup>1</sup>Papers which introduced some form of human capital as the source of growth are Uzawa (1965), Razin (1971), Romer (1986), Azariadis and Drazen (forth.), Barro (1988), King and Rebelo (1988) and Lucas (1988).

<sup>2</sup>Mainly the work originated by Mincer and Becker in the late fifties and sixties and applied extensively later (Becker, 1975 and Mincer, 1974). See also Ben-Porath (1967), Heckman (1976) and Rosen (1976).

anced growth path. The set up is expected to be “realistic”. By “realistic” we mean a framework able to resemble key aspects of a real economy. Aggregate measures of growth, saving rate and interest rates as well as individual age profiles of major life-cycle variables —mainly labor-training decisions as well as consumption-savings decisions— must be consistent with real economies. The term “realistic” requires some qualifications regarding the role of life-cycle savings in the light of the recent debate on the motives of capital accumulation. The next section is devoted to that issue.

In section 3 the basic framework is described. Our model extends the simulation model developed by Auerbach and Kotlikoff (1987)<sup>3</sup>. The AK model was designed to address, in a dynamic framework, expected and unexpected switches in fiscal policy for the U.S. economy. It includes neither growth nor endogenous human capital decisions<sup>4</sup>.

Unlike infinite horizon versions of the time-allocation model of human capital investments<sup>5</sup>, in a nonaltruistic life-cycle model it is necessary to assume some form of spillover effects in order to transfer “relevant knowledge” from generation to generation. The fundamental assumption of this paper comes from Azariadis and Drazen (forth.) who assume that every new generation is endowed with the average

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<sup>3</sup>Hereafter referred as the AK Model.

<sup>4</sup>The AK model was extended in Auerbach et. al. (1989) to account for exogenous technological change and growth. Our purpose is to include training decisions and endogenous growth.

<sup>5</sup>Uzawa (1965), Razin (1971), Lucas (1988).

level of skills prevailing at birth date. Unlike Azariadis and Drazen's work, we concentrate on unique equilibrium growth rates.

In section 4 we simulate the model and study the link between the rate of long-run per capita growth with the population growth rate and policy parameters. The model provides a new explanation of the inverse relation between the rate of population growth and rate of per capita growth. Fertility models assume that the relation resides at the household level and that it relies on the trade-off that parents face between quantity and quality of children. In these models, the direction of causality goes from an exogenous technical change growth to the endogenous population growth rate. In our model the rate of population growth and the rate of economic growth are related by the extent to which the first affects the magnitude of the spillover effect that causes growth, and the direction of causality goes from the exogenous population growth to the endogenous income per capita growth.

The inverse relation between population growth and income growth per capita is a remarkable result in this class of models. The infinite horizon framework gives a direct, and therefore counterfactual, relation between these two variables.

The ultimate objective of individuals is to maximize their welfare which only depends on consumption and leisure. Household's investments in physical and human capital is solely motivated by the desire to forgo consumption and leisure. Taxation policy can affect the proportion of these "assets" in total households wealth. Tax-

ation policy which favors the accumulation of human capital can positively affect the long-run rate of income per capita growth.

In a final section the main results are summarized.

## **2 The Choice of a Life-Cycle Model for Savings**

Our life-cycle model is a general equilibrium intertemporal model which integrates the Life-Cycle theory of savings with the Life-Cycle theory of human capital and labor supply. In their controversial paper, Kotlikoff and Summers (1981) have challenged the dominant role that the Life-Cycle theory of savings has played in explaining accumulated wealth<sup>6</sup>. Although the debate is not settled yet<sup>7</sup>, it seems apparent that intergenerational transfers play an important role in explaining aggregate wealth.

Our model, however, requires a theory of savings to be closed. Neglecting the role of intergenerational transfers has two justifications. First, it does not seem to be a consensual theory of intergenerational transfer and savings as yet, and second, the debate is on the level of wealth and not on the change of wealth through time. As argued below, any alternative to "hump savings" would lead to similar results. Let us take one point at a time.

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<sup>6</sup>The theory is associated to the names of Ando, Brumberg and Modigliani. See Ando and Modigliani (1963).

<sup>7</sup>For a good summary of the debate see Kotlikoff (1988) and Modigliani (1988).

The main alternative to savings for retirement was thought to be the altruistic bequest model first developed by Barro (1974). The evidence from micro as well as aggregate data is probably leaning against the major implications of the altruistic model of bequests. Recently, two powerful tests from micro data have been developed by Abel and Kotlikoff (1989) and Altonji, Hayashi and Kotlikoff (1989). The Abel-Kotlikoff test exploits the Ricardian implication that consumption from all cohorts (age groups) should move together. The test rejects the implication when it is performed in first differences. Altonji, Hayashi and Kotlikoff use a sample that allows to study direct information from parents and offsprings in order to verify whether the members of the same "dynasty" do share the same budget constraint. The results are also largely in support of the view that the relevant decision-making unit is the finite-lived household (life-cycle theory) as opposed to the infinite-lived dynasty (altruism model). On the other hand the strong neutrality implications from altruistic models seem to be inconsistent with the aggregate evidence on the impact of policy. The debate on the Ricardian Equivalence seems to be leaning against the implications of the altruistic view<sup>8</sup>. The role of intergenerational transfers, however, is important and new research emphasizes involuntary bequests as the result of precautionary savings under incomplete annuity markets<sup>9</sup>. Meanwhile,

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<sup>8</sup>See Bernheim (1987) and "Symposium on the Budget Deficit", *The Journal of Economic Perspective*, Spring 1989, vol. 3, no. 2.

<sup>9</sup>See Kotlikoff (1988) for more on these lines.



the well known savings for retirement seems to be the natural benchmark.

The second reason for using the life-cycle is that our result does not depend on the level of aggregate wealth, which is the aspect of the life-cycle model questioned by Kotlikoff and Summers (1981). As mentioned in the introduction, our purpose is to have a growth model which satisfies the condition of a fixed capital-income ratio along the balanced growth path. Therefore, regardless of the motives for holding a given volume of physical wealth, this must grow at the same rate that income does. After identifying the “engine” of growth, which in this model is *human* capital accumulation, *physical* capital accumulation will catch up with the pace of the “engine” and it is not a source of long-run growth per se.

### 3 The Framework

The purpose of this section is to introduce endogenous training decisions into the AK model. In order to keep things simple we limit the scope of the AK model in the following aspects. First, we are interested in learning how a steady growth rate depends on other parameters, including policy parameters and population growth, so we will not study transitional paths. Second, we will consider only income and consumption taxes which the government uses to finance its own consumption at a fixed proportion of GNP (neither national debt nor budget deficits are present). Both limitations could be easily relaxed.

### 3.1 The Individual's Problem

Individuals have a lifespan from adult-age one through fifty five (actual age 21 through 75). In what follows we describe the optimization problem for the particular cohort age 1. The individual maximizes a time separable utility function of the form

$$U = \frac{1}{1 - 1/\gamma} \sum_{t=1}^{55} (1 + \delta)^{-(t-1)} u_t^{(1-1/\gamma)} \quad (1)$$

where the subutility  $u_t$  is of the CES form

$$u_t = (c_t^{1-1/\rho} + \alpha l_t^{1-1/\rho})^{\frac{1}{1-1/\rho}} \quad (2)$$

and  $c_t$  and  $l_t$  are consumption and leisure respectively.

The flow budget constraint can be expressed as

$$A_{t+1} = [1 + r_t(1 - \tau_y)]A_t + w_t e_t(1 - l_t - h_t)(1 - \tau_y) - (1 + \tau_c)c_t \quad (3)$$

where besides the variables already defined,  $r_t$  is the interest rate,  $w_t$  is the wage per efficiency unit,  $e_t$  represents the units of human capital (or efficiency units of labor) per individual,  $h_t$  is time devoted to training (or to human capital accumulation) and  $\tau_y$  and  $\tau_c$  are the proportional tax rates of income and consumption respectively.

The optimal choice must satisfy the non negative constraint for working time and for training time<sup>10</sup>. Therefore, for all  $t = 1, \dots, 55$

$$1 - l_t - h_t \geq 0 \quad (4)$$

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<sup>10</sup>From inspection of the utility function, it can be noticed that no optimal solution exists for non positive leisure  $l_t$ , therefore the non negativity constraint for leisure will not be considered explicitly.

$$h_t \geq 0 \quad (5)$$

The individual faces a technology on human capital  $e_{t+1} = \bar{g}(h_t, e_t)$  which defines the profile of human capital as a function of time invested in training and the stock of human capital. For convenience we define this technology to be homogeneous of degree 1 in  $e_t$ , and therefore

$$e_{t+1} = e_t g(h_t) = e_1 g(h_1) \cdots g(h_t) \quad (6)$$

where  $g'(h) > 0$  and  $g''(h) < 0$ , and  $e_1$  is endowed at birth. We postpone the discussion on the spillover needed to sustain growth in order to concentrate on the individual's choice.

Given  $\{r_t, w_t, \tau_y, \tau_c\}$ , for the relevant horizon, the individual chooses paths of  $\{c_t, l_t, h_t, A_{t+1}\}$  for  $t = 1, \dots, 55$ , in order to maximize (1)-(2) subject to (3)-(6). The initial and terminal condition  $A_1 = 0$  and  $A_{56} = 0$  must be satisfied (no bequests). To solve the problem, define  $\lambda_t$  as the lagrange multipliers for budget constraint (3) and  $\mu_t$  and  $\epsilon_t$  as the (dollar-valued) lagrange multipliers for constraints (4) and (5) respectively. The first order conditions of the individual for  $t = 1, \dots, 55$  are

$$(1 + \delta)^{t-1} \Omega_t c_t^{-1/\rho} = (1 + \tau_c) \lambda_t \quad (7)$$

$$(1 + \delta)^{t-1} \Omega_t \alpha l_t^{-1/\rho} = w_t^* \lambda_t \quad (8)$$

$$[1 + r_{t+1}(1 - \tau_y)] \lambda_{t+1} = \lambda_t \quad (9)$$

$$\lambda_t (w_t^* - \epsilon_t) = \sum_{j=t+1}^{55} \lambda_j (1 - l_j - h_j) (1 - \tau_y) w_j \frac{\partial e_j}{\partial h_t} \quad (10)$$

where

$$\Omega_t = \left( l_t^{1-1/\rho} + \alpha l_t^{1-1/\rho} \right)^{(\rho-\gamma)/(1-\rho)\gamma} \quad (11)$$

$$w_t^* = w_t e_t (1 - \tau_y) + \mu_t \quad (12)$$

Eliminating  $\lambda_t$  and after some manipulation, (7)-(10) can be expressed as

$$c_{t+1} = \left( \frac{1 + r_{t+1}(1 - \tau_y)}{1 + \delta} \right)^\gamma \left( \frac{v_{t+1}}{v_t} \right) c_t \quad (13)$$

$$l_{t+1} = \left( \frac{1 + r_{t+1}(1 - \tau_y)}{1 + \delta} \right)^\gamma \left( \frac{v_{t+1}}{v_t} \right) \left( \frac{w_{t+1}^*}{w_t^*} \right)^{-\rho} l_t \quad (14)$$

$$l_t = \left( \frac{w_t^*}{\alpha(1 + \tau_c)} \right)^{-\rho} c_t \quad (15)$$

$$(w_t^* - \epsilon_t) = \sum_{j=t+1}^{55} \left[ \Pi_{s=t+1}^j 1 + r_s(1 - \tau_y) \right]^{-1} (1 - l_j - h_j)(1 - \tau_y) w_j \frac{\partial e_j}{\partial h_t} \quad (16)$$

where

$$v_t = \left[ 1 + \alpha^\rho \left( \frac{w_t^*}{1 + \tau_c} \right)^{(1-\rho)} \right]^{(\rho-\gamma)/(1-\rho)} \quad (17)$$

Equations (13)-(15) are the same first order conditions from the AK model.<sup>11</sup> Equation (16) however needs some additional comments. This expression is the arbitrage condition for human capital investments. Suppose that we have an interior solution so  $\mu_t$  and  $\epsilon_t$  are zero. The left-hand side is the value of the last hour invested in working. By the end of period  $t$ , the individual receives the wage for the last hour worked. Alternatively, the individual could use the last hour to increase his human capital for next period. The change in next period human capital ( $\frac{\partial e_{t+1}}{\partial h_t}$ )

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<sup>11</sup>The reader is referred to Auerbach and Kotlikoff (1987), chapter 3, for more details.

times the wage times the number of hours worked during next period represents the next period "cash flow" of the investment. However the last hour invested today not only increases next period human capital, but also the human capital stock for all remaining years (see expression 6). The value of the last hour invested in training in period  $t$  is the present value of the cash flow related to that investment. This is the RHS in (16), which must match the opportunity cost of human capital investments, i.e. working. Furthermore, if  $\epsilon_t$  is positive ( $h_t = 0$ ), then the value of the last hour worked could be bigger than the present value of the last hour invested in training, as no more hours can be taken away from training for arbitrage purposes. On the contrary, if the individual is retired from working activities ( $\mu$  positive), then the yield on human capital investment could be bigger as the opposite is true.<sup>12</sup> By virtue of the functional form (6), which is homogeneous of degree one (HD1) in  $e_t$ , (16) can be conveniently expressed as  $(w_t^* - \epsilon_t)/PV E_t = g'(h_t)/g(h_t)$ , where  $PV E_t$  is the present value of labor earnings from period  $t + 1$  up to the end of life.

For a given  $c_1$  and  $e_1$ , (13)-(17) solve the paths for consumption, training and leisure. Furthermore the multipliers  $\mu_t$  and  $\epsilon_t$  must satisfy the Kuhn-Tucker conditions  $(1 - l_t - h_t)\mu_t = 0$  and  $h_t\epsilon_t = 0$  respectively. Finally integrating (3) using initial and terminal conditions  $A_1, A_{56} = 0$ , we can find the optimum solution for  $c_1$ .

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<sup>12</sup> Alternatively consider the "cash flow" of last hour worked equal to the shadow wage, which includes  $\mu_t$ .

The above paragraph fully describes the individual solution.

### 3.2 The Firm's Problem

Assuming many atomistic firms with identical technologies of production, we can express the firm's problem in aggregate terms. Firms hire nondepreciating capital and effective units of labor, until factor prices and marginal rates of substitution are equalized.

Firms face the Cobb-Douglas technology

$$Y_t = BK_t^{1-\beta} L_t^\beta \quad (18)$$

where  $K_t$ ,  $L_t$  and  $B$  are aggregate physical capital, effective labor and a technological parameter respectively.

The first order conditions for the firm's problem are

$$w_t = \beta B \left( \frac{K_t}{L_t} \right)^{1-\beta} \quad (19)$$

$$r_t = (1 - \beta) B \left( \frac{K_t}{L_t} \right)^{-\beta} \quad (20)$$

### 3.3 Equilibrium

Capital stock and effective labor supplied by households at period  $t$  are

$$K_t = \sum_{s=1}^{55} A_t^s (1+n)^{t+1-s} \quad (21)$$

$$L_t = \sum_{s=1}^{55} (1 - l_t^s - h_t^s) e_t^s (1+n)^{t+1-s} \quad (22)$$

where  $x_t^s$  means variable  $x$  at period  $t$  for cohort age  $s$ , and  $n$  is a fixed and exogenous rate of population growth. In (21)-(22) we have implicitly normalized the size of cohort age 1 at period 0 to be equal to 1.

**Definition.** A sequence  $\{w_t, r_t\}$  represents an equilibrium if it satisfies the individual optimization problem (12)-(17), the firm problem (19)-(20) and the equilibrium conditions (21)-(22). Furthermore, in a “steady growth” (balanced growth) equilibrium the sequence  $\{w_t, r_t\}$  is constant and equal to  $(\bar{w}, \bar{r})$  for all  $t \geq 0$ .

### 3.4 Human Capital Production and Growth

The final piece that completes the story is the specification of the human capital technology. This technology plays the dual role of shaping the profile of individual variables over the life-cycle and being the “engine” of aggregate economic growth.

Heckman (1976) estimated a function of the form  $e_{t+1} = a(h_t e_t)^b + (1 - d)e_t$ . The estimates are not precise, with parameters not significantly different than zero at usual level of significance, with the only exception of  $b$ . Probably the most challenging difficulties of these type of estimations is the lack of appropriate data. In Heckman’s words, “reported hours of work may include investment time if such investment takes place on the job. Thus, caution is required in directly relating the theoretical construct to actual data”<sup>13</sup>. The second problem is related to the functional form. Like most authors in the field, Heckman uses a functional form

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<sup>13</sup>Heckman (1976), page S14.

with the slope going to infinite as hours invested go to zero. This implies that individuals do not stop investing throughout the working life (the yield on human capital investments can be big enough by investing small enough amount of hours). We relax this implication below. Finally the functional form is not HD1 in  $e_t$  which poses some problems for the numerical solution of the model.

For the reasons above we choose an arbitrary function for (6), where  $g(\cdot)$  takes the parabolic form

$$\frac{e_{t+1}}{e_t} = g(h_t) = 1 - d - v_1 h_t^2 + v_2 h_t \quad (23)$$

The slope is finite at zero training and the functional form is strictly concave. The depreciation term  $d$  is common to most life-cycle models of training.

Lucas (1988) follows closely Uzawa (1965) and uses a linear function  $g = 1 + \nu h$ . In Lucas' infinite horizon model, this formulation is enough to sustain growth per capita. A constant training time  $h$  implies a permanent rate of growth of households human capital at a rate  $\nu h$ . In an overlapping generations model, however, individuals die with their embodied human capital. To sustain growth per capita, we need to assume some form of spillover so that the relevant "social knowledge" is transferred from generation to generation. Azariadis and Drazen (forth.) assume that every individual is born with the average level of skills around at the date he is born.

In order to implement Azariadis and Drazen's assumption for our framework,



we define the average human capital of this economy as  $\bar{e} = L/H$ , where  $L$  is total labor supplied,  $H$  is the total supply of hours from households and  $\bar{e}$  is the average human capital. Using (22),  $\bar{e}$  at period  $t$  can be expressed as

$$\bar{e}_t = \frac{\sum_{s=1}^{55} (1 - l_t^s - h_t^s) e_t^s (1 + n)^{t+1-s}}{\sum_{s=1}^{55} (1 - l_t^s - h_t^s) (1 + n)^{t+1-s}} \quad (24)$$

The assumption that relates the human capital endowment of new generations to the average human capital that prevails at the time of birth can be expressed as

$$e_{t+1}^1 = \bar{e}_t \quad (25)$$

If the process of human capital accumulation leads to a growing path of the average of human capital, the economy would keep growing as new generations are endowed with a higher level of human capital than older generations. Furthermore, for a time-invariant age-profile of working time and training (an implication of our model), the balanced rate of growth per capita is equal to the rate of growth of  $\bar{e}$ <sup>14</sup>. Finally, the “equilibrium” rate of growth is endogenous. The extent to which it depends on technological, policy and demographic parameters is the purpose of the next section.

Before that, an important implication of our assumption in (25) should be emphasized. Notice that the spillover effect does not affect the technology available to

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<sup>14</sup>Defining this rate as  $x$ , from (22) aggregate labor grows at  $(1 + n)(1 + x) - 1$  which must be the rate of growth of aggregate capital and income in a balanced growth path. Therefore  $x$  is also the rate of growth of income per capita.

those who generate the spillover. As individuals do not care about the utility of their descendants, the spillover does not generate an externality in the sense of a market failure or a suboptimal equilibrium. Unlike previous models with externalities, the government cannot implement a pareto improving scheme.

## 4 Simulation Results

### 4.1 Parameterization of the Model

The first step consists in calibrating the model. The task is fairly simple as the AK model is already very well calibrated for the U.S. economy. We rely on the same values for most of the parameters with only a few exceptions.<sup>15</sup> We summarize and explain the changes below.

*Taxes, Utility and Production Function Parameters.* The baseline simulation in the AK model uses  $\tau_y = 0.15$ ,  $\tau_c = 0$ ,  $\gamma = 0.25$ ,  $\rho = 0.8$ ,  $\alpha = 1.5$ , and  $\beta = 0.75$ .

The first change restricts the subutility function (2) to the Cobb-Douglas case, that is  $\rho = 1$ . This is a requirement of our endogenous model of growth if we want to have a balanced growth path with a constant interest rate. The reason is that in a growth model the endowment of consumption goods (human capital) is growing while the endowment of leisure is fixed. For a different-than-one intratemporal elasticity of substitution between consumption and leisure, the wage per efficiency

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<sup>15</sup>For the empirical background of the choices in the AK model see Auerbach and Kotlikoff (1987), chapter 4.

unit of labor would not be fixed at a balanced growth path, and from (19)-(20) the interest rate cannot be fixed either. As an alternative to this restriction, Auerbach et. al. (1989) assume that technical change is "time-augmenting", increasing the endowment of leisure and consumption at the same exogenous rate. If technical change is exogenous, that assumption preserves the generality of the utility function, but with endogenous growth, it would be arbitrary to assume that the endowment of leisure grows precisely at the same rate.<sup>16</sup> Once we choose the Cobb-Douglas form  $u_t = c_t^\alpha l_t^{1-\alpha}$ , then the choice for  $\alpha = 1.5$  in the general form (2) becomes  $\bar{\alpha} = 0.4$  for the Cobb-Douglas. Finally the Cobb-Douglas assumption implies that all cohorts, regardless of initial endowment in human capital, will choose the same age profile of leisure. Also, if the human capital technology is HD1 in  $e_t$  the age profile of working and training hours will also be cohort-invariant. This simplifies very much the numerical solution of the model.

The second deviation from the AK parameterization is to increase  $\gamma$  to be 0.5. The reason is that this value for  $\gamma$  leads to a realistic profile of individual consumption, which grows at about 2.3% per year.<sup>17</sup> Besides, the longitudinal profile of

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<sup>16</sup> A way to make compatible the more general time-augmenting approach with endogenous growth would be to assume that leisure is a household-produced commodity which uses household time and consumption as inputs. Assume that leisure is produced by  $l^* = f(l, c)$  where  $l$  is time and  $c$  is consumption. If  $f$  is HD1 in  $c$ , then consumption and leisure can grow at the same rate for a fixed time  $l$ . Consumption could also be an "external" effect on leisure, avoiding the need to deviate real resources from consumption. An alternative approach, with similar consequences, would be to use Heckman's (1976) assumption, who introduces the stock of human capital in the utility function multiplying or "augmenting" leisure, that is  $f(\cdot) = l e$ .

<sup>17</sup> Consistent with Kotlikoff and Summers' (1981) finding.

leisure which arises from  $\gamma = .25$  is too flat and does not leave enough "room" for the training investment which take place early in life.<sup>18</sup>

Finally we normalize  $B = 1$  in (18), which arbitrarily fixes the unit of account. *The Human Capital Technology.* Apart from the estimation by Heckman (1976) described above, there is not much evidence which can be translated into our time-investment technology. Most of the human capital empirical literature has been devoted to understand the relation between year of schooling and earnings.<sup>19</sup> Besides the data problems mentioned above, a sensible model of long-run growth should have a broadly defined technology of human capital. It should at least include R&D, which is supposed to be an important component of human capital investments. This of course complicates even more the translation of empirical estimations into our human capital technology. Finally, the simultaneous dimension of training-leisure-working decisions has not been approached empirically.<sup>20</sup> This means that we have to rely on sensitivity analysis in order to find sensible values of the parameters  $v_1$ ,  $v_2$ , and  $d$  in our human capital technology.

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<sup>18</sup> Estimates for this parameter range between 0.1 and more than one. See Auerbach and Kotlikoff (1987), pages 50-51.

<sup>19</sup> For a survey see Rosen (1975). An important part of this literature has been devoted to the problem of controlling by ability (Behrman et. al., 1980).

<sup>20</sup> Regarding this point, Rosen (1975) suggests that the simultaneity problem "may require the unhappy prospect of combining numerical solutions and estimations". This paper contributes to the unhappy but necessary side of the equation.

## 4.2 The Solution Method

We employed a Gauss-Siedel algorithm very similar to the one developed by Auerbach and Kotlikoff (1987, ch. 4).<sup>21</sup> Given initial guesses for capital and labor, from (19)-(20) we compute wage and interest rate. Then we iterate the individual's first order conditions to find a solution to all individual variables (including shadow prices) for cohort age 1. Next we aggregate individual choices for every cohort. The time distribution over the lifespan is invariant across cohorts, but we need to guess the rate of income per capita growth as this rate "scales down" the endowment of older cohorts. Finally (21), (22) and (24) give us an update of capital, labor and the rate of economic growth respectively. Then we use an interpolation between the old and new values of capital, labor and per capita growth and repeat the procedure above. The algorithm converges when all individual variables—for all years of the lifespan—as well as capital, labor and per capita growth satisfy a convergency criterion.

## 4.3 Sensitivity Analysis: The Human Capital Technology

Table 1 summarizes a set of parameters for the human capital technology which yield sensible rates of growth per capita, capital-income ratio, before-tax interest rates and saving rates. The rate of growth per capita fluctuates between 1.9%

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<sup>21</sup>I am indebted to Alan Auerbach and Larry Kotlikoff for giving me their computer algorithm which was crucial to develop the algorithm for this paper.

	$v_1$	$v_2$	$d$	$\frac{K}{Y}$	Saving Rate (%)	Interest Rate (%)	Wage	Growth per capita (%)
1	0.09	0.18	0	3.7	10.8	6.8	1.15	1.9
2	0.09	0.18	0.005	4.0	10.2	6.2	1.19	1.5
3	0.09	0.18	0.01	4.5	9.3	5.6	1.24	1.0
4	0.08	0.16	0	4.2	9.2	5.9	1.21	1.2
5	0.08	0.16	0.005	4.7	8.1	5.3	1.26	0.7
6	0.08	0.16	0.01	5.3	6.6	4.7	1.31	0.2
7	0.09	0.16	0	4.3	8.9	5.8	1.22	1.1
8	0.09	0.16	0.005	4.8	7.7	5.2	1.27	0.6
9	0.09	0.16	0.01	5.5	6.1	4.6	1.32	0.1

Table 1: Sensitivity Analysis of Human Capital Technology

and 0.1%, the capital-income ratio fluctuates between 3.7 and 5.5 and the interest rate fluctuate between 6.8% and 4.6%. The saving rate (as % of NNP) is normal for OECD standards, but it appears high compared to the simulations in the AK model.<sup>22</sup> Auerbach and Kotlikoff (1987) attribute the low saving rate to the above-mentioned inability of a life-cycle model to explain aggregate wealth. In a growth model, however, a life-cycle model can explain a higher saving rate simply because a higher proportion of aggregate income is in young people's hands, without the need to rely on unrealistic rate of population growth or consumption profile. Setting the growth rate to zero would yield a saving rate of 3.6% in first row of Table 1 and 4.5% in third row. This shows that the higher saving rate in our model is due to

<sup>22</sup>The saving rate is about 3.7% in the base simulation of the AK model. The U.S. rate average 7.9% since 1950.

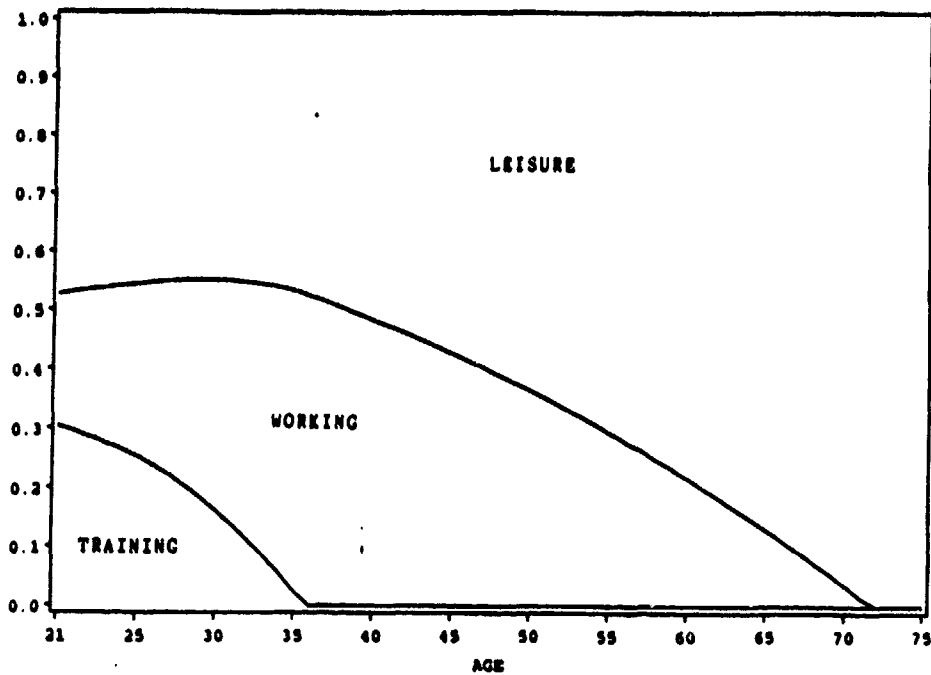


Figure 1: Distribution of the Time Endowment Over the Life Cycle

growth.

Figure 1, 2 and 3 show typical age profiles of the main individual variables, using the simulation from the first row in Table 1. Figure 1 show the distribution of time endowment over the life cycle. The individual invests about 30% of time in the beginning of adult-life and decreases the investment to zero by age 36. Working time increases from about 20% of total time early in life to peak at age 36 and decreases from there on. Finally leisure stays flat early in life to decrease after that. Full-time retirement takes place 3 years before dying but partial retirement much earlier. The human capital technology used here does not seem to be powerful enough to produce full-time schooling (non working time) early in life.

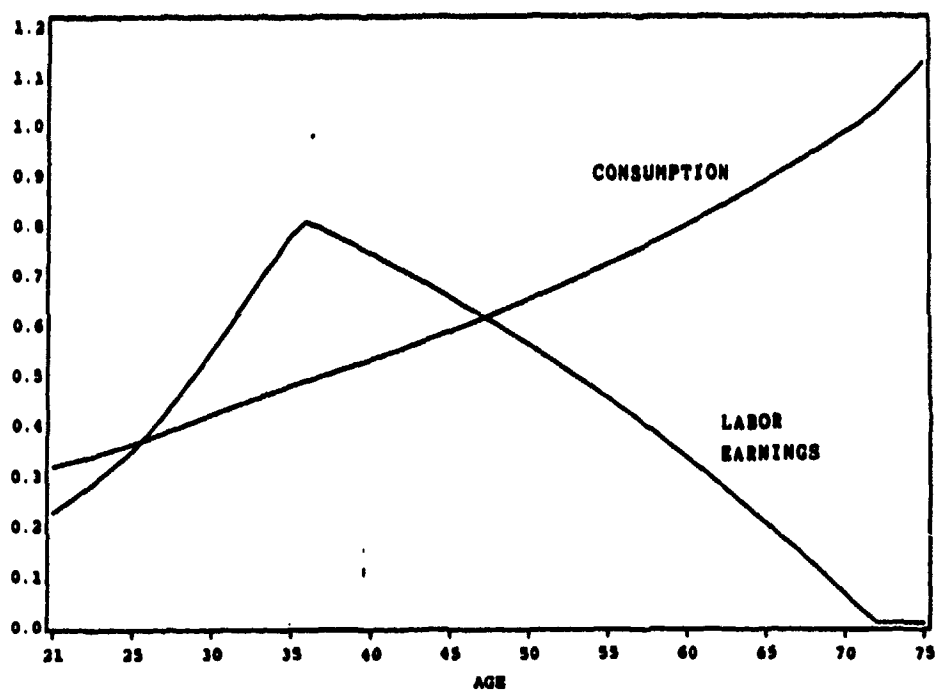


Figure 2: Longitudinal Age-Profile of Consumption and After Tax Labor Earnings

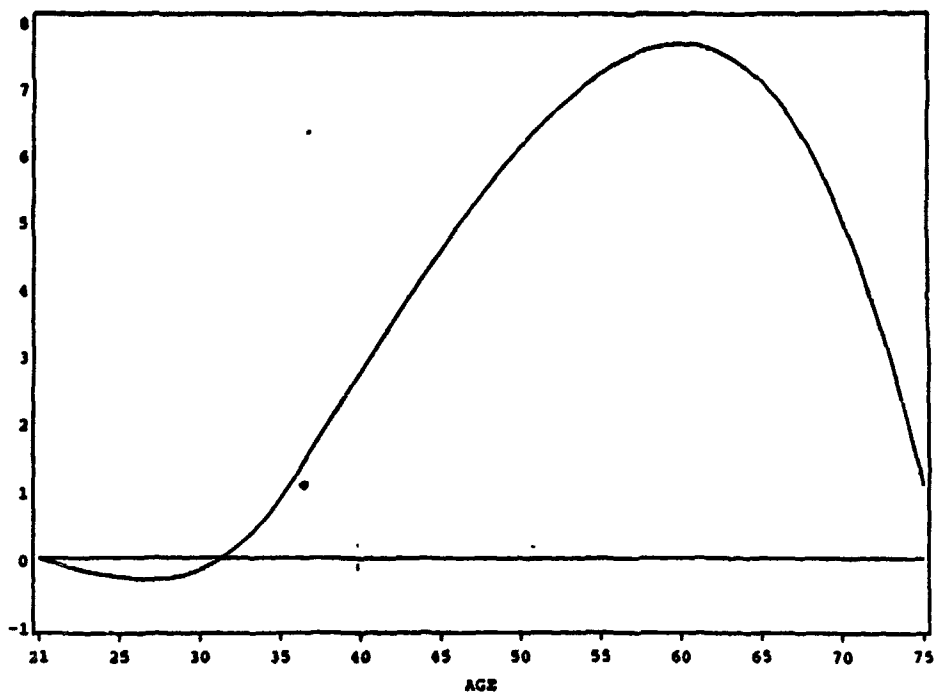


Figure 3: Longitudinal Age-Profile of Asset Holdings



Figure 2 shows the resulting longitudinal profile of consumption and after-tax labor earnings. The profile of consumption grows at 2.3% (as found by Kotlikoff and Summers, 1981) although the profile of earnings seems to peak early.<sup>23</sup> As discussed in section 2, however, we were expecting a strong life-cycle behavior in order to generate enough savings. The lack of smoothness of the earnings profile is due to the human capital technology. We could smooth out the earnings function if we were to include an exogenous age-dependent quadratic term in the human capital technology, as included in most empirical works on human capital.

Figure 3 show the longitudinal profile of asset holdings. We can see that debt appears the first 10 years and that the profile peaks at about age 60.<sup>24</sup>

Summing up, we can say that the human capital technology provides a reasonable representation of life-cycle behavior. The human capital technology can generate training investments early in life, and an increasing and then decreasing profile of working time. Moreover, the simulations provide a good profile of consumption and aggregate measures of growth, interest rates and key ratios such as the capital-income ratio and the saving rate.

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<sup>23</sup>Evidence from Kotlikoff and Summers (1981) suggest that labor earnings and consumption go very close early in life and that life-cycle savings arise late in life.

<sup>24</sup>The fact that debt arises early in life makes the model appropriate to study the impact of borrowing constraints. This could be done by forcing individual to hold a nonnegative amount of assets at every point in time.

Population Growth (%)	Income Growth per capita (%)	
	d = 0	d = 0.01
0	2.0	1.1
1	1.9	1.0
2	1.8	0.9
4	1.6	0.6

Table 2: Population and Income Per Capita Growth

#### 4.4 Population Growth and Economic Growth

In this part we study the effect of an exogenous rate of population growth on the endogenous rate of income growth per capita. The results are shown in Table 2 for the parameters in first and third row in Table 1 (zero and 1% depreciation in human capital technology respectively). As we can see in Table 2, there is an inverse relation between the rate of population growth and the rate of economic growth, which is consistent with the evidence<sup>25</sup>.

The rate of population growth affects the age distribution of the country's population, and because of that, it also affects the equilibrium capital intensity, interest rate and the optimum choice of training time for all individuals. Therefore, two independent effects are at work. First, for a given cross-section profile of human

<sup>25</sup>Evidence on this negative relation is in Tamura (1987) and Collins (1989). Collins actually provides evidence that directly relates the income growth per capita and the average age of the population. The rate of population growth and the average age of the population are inversely related in the long-run.

capital, interest rate and capital intensity, an increase in the number of individuals with low human capital will reduce the rate of growth, as the latter depends on the rate of change of the average human capital accross individuals. Second, the population growth rate perturbs the general equilibrium of the economy (capital intensity and interest rate) and therefore the optimum choice of training time for all individuals. In general both effects could work in favor or against growth, depending the result on the particular fundamental and demographic structure of the economy. From Table 2 we can say that for our calibration the impact of population is against growth.

Of the two effects mentioned above, the first will be emphasized in this section. The second is studied in a two-period model in an Appendix.

To see the impact of the population growth rate in detail, we decompose the human capital stock by age group. We do so for the case of zero depreciation (first column in Table 2) and the results are presented in Table 3. Let us clarify what we mean by average human capital by age group. We simply apply the formula (24) for every age group. Note that by average human capital we do not mean the average of individual' stock of human capital, but we weight every individual by the number of hours he/she works. Strictly speaking, by average human capital we mean a normalized version of labor earnings per hour for every age group. This definition of average human capital makes the most sense if we want an old retired

Age Group	n = 1%		n = 2%		n = 4%	
	Population (%)	Average Human Capital (%)	Population (%)	Average Human Capital (%)	Population (%)	Average Human Capital (%)
21-30	22	1.113	27	1.103	37	1.083
31-40	20	1.156	22	1.121	25	1.057
41-50	18	0.976	18	0.947	17	0.908
51-60	17	0.807	15	0.792	11	0.782
61-75	22	0.675	18	0.673	10	0.687
Total	100	1.019	100	1.018	100	1.016

Table 3: Distribution of Population and Average Human Capital by Age Group

“genius” to contribute nothing to today’s growth. This does not mean that today’s growth has nothing to do with him or her, but that his/her contribution is already internalized in the level of human capital of the currently working population.

Due to our normalization of one unit of human capital for cohort age 1 at period 0, the average of human capital for all the economy in the last row of Table 3 is also the factor of economic growth. We can see that the cross-section profile of average human capital increases with age and then decreases. It peaks at age 35. When the population growth increases, the average age of the population decreases. For a given profile of human capital, the total average aggregate human capital (rate of income per capita growth due to the normalization) decrease (increases) if the “relevant” portion of the cross-section profile is the increasing (decreasing) portion.

To understand the importance of the slope of the cross-section profile of human capital, we combine the equations (6), (24) and (25) for a two-period version of the model without leisure and with population age 2 at period  $t$  normalized to be equal to 1. Then we have

$$\bar{e}_t = e_{t+1}^1 = \frac{(1+n)(1-h)e_t^1 + e_{t-1}^1 g(h)}{(1+n)(1-h) + 1} \quad (26)$$

where  $h$  is training time invested at age 1. Recalling that  $1+x = e_t^1/e_{t-1}^1$ , where  $x$  is the rate of income per capita, we can express the above expression as

$$1+x = \frac{(1+n)(1-h) + g(h)/(1+x)}{(1+n)(1-h) + 1} \quad (27)$$

The slope of the longitudinal profile of human capital is  $g(h)$  and  $g(h)/(1+x)$  is the slope of the cross-section profile of human capital. If the latter is positive, then an increase in  $n$  increases the number of low-human capital individuals and the equilibrium value for  $x$  will be reduced. Of course this requires that the optimum choice for  $h$  be reduced (Appendix). From (27) we can see that a direct relation between population and economic growth will arise if the cross-section profile of human capital is negative. In our large-scale version the cross-section profile increases and then decreases. The "relevant" portion that we made reference above will depend on how young is the population on average. This suggests that the inverse relation between population and economic growth is not monotonic. For very low rates of population growth (presumably negative rates) the relation could be positive.

Now it should be clear what is going on in Table 3. The fact that the whole profile shifts downward with population growth simply reflects the facts that the profile grows at a lower rate due to the negative effect of the population growth rate on the spillover effect that causes growth. The reader is referred to the Appendix for a discussion of how the optimum choice for  $h$  is affected, even when the direct effect described here is not at work.

The above result provides an alternative to recent endogenous fertility models to explain the observed inverse relation between economic and population growth<sup>26</sup>. This endogenous fertility literature assumes that the population responds endogenously to the economic environment. Parents would face a tradeoff between quantity and quality of children. In this literature the causality goes from an exogenous rate of technological change to an endogenous rate of population growth. In our model the causality goes from an exogenous rate of population growth to the endogenous rate of income per capita growth.

The above result deserves to be remarked. The traditional infinite horizon model of endogenous growth and exogenous population growth, first developed by Uzawa (1965) and extended by Lucas (1988)<sup>27</sup>, has the counterfactual implication that population growth and income growth per capita are directly related<sup>28</sup>. An in-

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<sup>26</sup>Barro and Becker (1989) and Tamura (1987) are models of this type.

<sup>27</sup>Lucas includes Romer's (1986) externalities into the Uzawa framework

<sup>28</sup>The reader is referred to Lucas (1988), equations (21) and (26).

tuitive argument would be as follow. In the well known Ramsey-Cass-Koopmans framework, an increase in population growth reduces the capital-labor ratio (have in mind the Modified Golden Rule condition). This increases the return of capital relative to the return of labor (wage). This happens because labor is exogenously supplied and the relative price adjusts to accommodate a relatively more abundant factor. In the Uzawa-Lucas framework both factors are endogenous. The same (presumably “incipient”) movement in relative prices discourages working and encourages training. The counterfactual implication is evident as hours invested in training and growth are directly related.

#### 4.5 Taxation and Economic Growth

It is likely that the relation between taxation and economic growth is going to occupy a great deal of researchers' time in the immediate future (Aaron, 1989). In this section we explore that relation in our model. The first three rows of Table 4 show that neither the choice of taxation (income or consumption) nor the level of these taxes has any impact on the equilibrium rate of growth. This result contrasts with Barro (1988) who assumes that government expenditures (revenues here) generate an external effect on the aggregate production function. In Barro's model, therefore, the size of the government matters. Although we do not have that external effect, the size of the government would matter if we were to abandon the assumption of

Tax Rate	Investment and Saving Rate (% of NNP)	Govt. Exp. and Revenue (% of NNP)	Private Consump- tion (% of NNP)	Growth per capita (%)
No taxes	12	0	88	1.9
$\tau_y = 0.15$	11	15	74	1.9
$\tau_c = 0.20$	12	15	73	1.9
$\tau_y = 0.20, \tau_w = -0.0667$	10	15	75	2.0
$\tau_y = 0.10, \tau_w = 0.0667$	11	15	74	1.8
$\tau_y = 0.05, \tau_w = 0.1333$	11	15	74	1.7
$\tau_y = 0, \tau_w = 0.20$	12	15	73	1.6

Table 4: The Choice of Taxation and Economic Growth

a Cobb-Douglas formulation for the subutility (2). The direction of the effect will depend on whether  $\rho$  is assumed greater or smaller than one. For the time being we are interested in exploring an alternative road.

Both consumption taxation and income taxation do not affect the return of *human* capital relative to *physical* capital. Households therefore hold the two “assets” in the same proportion and the “average” human capital is not affected. Next we explore the effect of taxation when a proportion of either *physical* capital income (interest income) or *human* capital income (labor income) is taken out of the tax base. We do so by introducing a labor income tax,  $\tau_w$ . We can arbitrarily favor either type of income, by combining the income tax with a labor income tax or subsidy. To keep things clear, we combine this two taxes in a way that total revenue is



kept fixed at 15% of NNP<sup>29</sup>. The last four rows in Table 4 summarizes the results. The rate of growth is higher when the proceeds of *human* capital is subsidized and *physical* capital income takes a higher proportion of the tax burden. We would be tempted to conclude that lifting the double taxation on dividends could hurt the rate of economic growth when revenues are kept fixed. That conclusion, however, would require the additional assumption that the process of human capital accumulation is mostly realized at the household level, as opposed to the corporate level, which is something we do not know. We can say, nevertheless, that taxation policy that favors human capital accumulation increases the rate of economic growth per capita.

In closing this section a reflection on some issues from the development literature is appropriate. It has been a long tradition in development to sustain that the rate of economic growth depends on the saving rate. Collins (1989) finds evidence from ten developing countries, mostly from Asia, which support this view. The savings rate and the rate of income per capita growth seem to move together in her sample period and group of countries. This relation also appears in Table 1. For a given tax structure and population growth, if countries differ in technological parameters, the saving rate and growth are directly related. Attempting to exploit that relation, however, might not be a good objective. In Table 4 we can see that for a given

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<sup>29</sup>The taxes must satisfy the linear relation  $\tau_y = 0.15 - \beta\tau_m$

technological and demographic structure, taxation policy that increases the saving rate (subsidy to capital accumulation) would reduce the rate of growth per capita, as a higher tax burden on labor income makes less profitable to invest in training.

The above result sounds counterintuitive but it is not. By saving rate, we usually do not mean “total saving” rate, which should include the resources not consumed (and not produced) due to the time withdrawn from production to be devoted to training. A subsidy to capital accumulation increases the saving rate in *physical* capital but reduces the “saving rate” in *human capital*. In our human-capital-as-an-engine-of-growth model, the income per capita growth is proportional to the saving rate in human capital.

## 5 Summary and Conclusions

Most models of economic growth are infinite horizon models where the role of human capital in “shaping” life-cycle variable is neglected. This paper introduces training decisions in a life-cycle model in order to study simultaneously the roles of human capital in life-cycle behavior and as an engine of growth.

Perhaps the most important lesson of this paper is that all the knowledge on growth we have accumulated by studying aggregative models of growth could be greatly enhanced by studying models of growth at a more disaggregated level.

The crucial assumption of the model regarding growth is that new generations

are endowed with an initial endowment of human capital which is endogenously determined by decisions from previous cohorts (Azariadis and Drazen, forthcoming). The assumption does not produce a second best competitive balanced growth path, so the government cannot implement a pareto improving scheme.

The model reasonably resembles life-cycle behavior and aggregate variables such as the rate of income growth per capita, capital-income ratio, savings rate, interest rate, etc.

The model provides an explanation to the inverse relation between the rate of population growth and the rate of economic growth per capita. Our explanation does not rely on the choice between quantity and quality of children as proposed by recent fertility-based models, where the direction of causality goes from an exogenous rate of technological change to the endogenous rate of population growth. In our model the (exogenous) rate of population growth affects the age distribution of the country's population and perturbs the magnitude of the spillover effect which causes growth. The direction of causality goes from the exogenous population growth to the endogenous income per capita growth. Future work should attempt to endogenize both variables.

Taxation policy can also affect the equilibrium rate of growth per capita. Taxation policy that favors the proceeds of *human* capital investment as opposed to *physical* capital investments (e.g. subsidy to labor income) causes households to

invest in greater proportion in the “the engine of growth”, positively affecting the long-run rate of growth per capita.

It is important to mention two aspects neglected in this paper. The labor literature considers as an important component of the human capital technology the fact that human capital can be acquired in a “joint venture” with working (Rosen, 1972). This learning-by-doing assumption has been introduced in infinite horizon models of growth (Romer, 1986; Lucas, 1988) and could also be introduced in our life-cycle framework. Secondly, we neglect transitional effects from switching policy, which might be very misleading. While the long-run conclusions are not affected, the short-run rates of economic growth could be very different.

## Appendix

The purpose of this appendix is to study, in a two-period version of the model, how the population growth rate perturbs the general equilibrium of the economy. The model here is the basic model in Azariadis and Drazen’s (forth.), enhanced by introducing population growth, and we replicate Azariadis and Drazen’s procedure to show existence and uniqueness in this economy. There is no leisure and capital fully depreciates in one period. We also keep their version of the spillover effect. Rather than equation (25), they assume  $e_t^1 = \bar{e}_t$ . Contemporaneous human capital average affects contemporaneous age-one endowment. The difference is unimportant

in a large-scale version, but it is extremely convenient in the two-period model. Note that the analogue to (27) will be

$$1 = \frac{(1+n)(1-h) + \frac{g(h)}{1+x}}{(1+n)(1-h) + 1} \quad (\text{A.1})$$

which means that

$$1+x = g(h) \quad (\text{A.2})$$

and  $n$  no longer have an independent effect in the spillover equation. The reason is that now the cross-section profile of human capital is flat (slope equal to one), and the only impact of population growth will be the indirect effect on  $h$ .<sup>30</sup> In this way we can isolate the direct effect emphasized in the text, and can concentrate in the indirect effect.

Now we explain how the equilibrium  $h$  could be affected by  $n$ . The arbitrage condition between human capital and physical capital takes the form<sup>31</sup>

$$f'(k) = 1+r = g'(h) \quad (\text{A.3})$$

where  $f$  is the production function in intensive form and  $k$  is the equilibrium capital-efficiency-labor ratio. Time subscripts are dropped for intensive variables, as we are interested in describing the steady state equilibrium. Both  $f$  and  $g$  are concave functions so (A.3) describes a positive-sloped locus in the  $\{k, h\}$  plane. We call that locus AA in Figure A.1. Next we need to find the equilibrium in the saving-

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<sup>30</sup>The population growth rate will appear again in more than two periods, so this trick is valid only for expositional purposes.

<sup>31</sup>It could be directly obtained from (16) by assuming an interior solution.

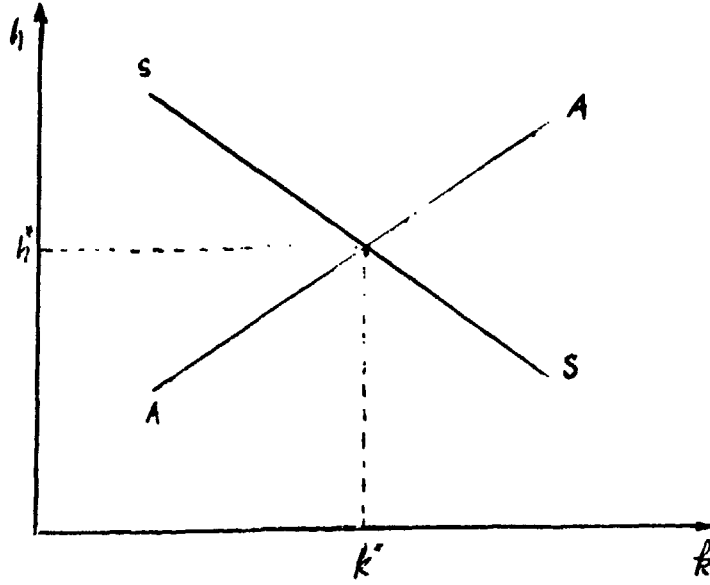


Figure A.1: Equilibrium of Capital Intensity and Training Time

investment equation. The equilibrium can be expressed as

$$K_{t+1} = N_t s[1 + r, we_t^1(1 - h), we_t^1 g(h)] \quad (\text{A.4})$$

where  $s$  is the saving function of young individuals,  $K$  is aggregate capital stock and  $N_t$  is number of young individuals at period  $t$ . Gross substitutability assures that the saving function is increasing in the first term. Homoteticity of the utility function assures that the saving function is linearly homogeneous in the second and third term. Finally the saving function is increasing in the second term and decreasing in the third. Noting that aggregate labor is  $L_{t+1} = N_{t+1}(1 - h)e_{t+1}^1 + N_t g(h)e_t^1$ , and

using the above properties, we can express (A.4) as

$$[(1+n)(1-h)\frac{e_{t+1}^1}{e_t^1} + g(h)]\frac{k}{w} = s[f'(k), (1-h), g(h)] \quad (\text{A.5})$$

Finally, recalling that the ratio of first period endowments is  $1+x$  and using (A.2), (A.5) becomes

$$\frac{k}{w(k)} = s[f'(k), \frac{(1-h)}{g(h)[(1+n)(1-h)-1]}, \frac{1}{(1+n)(1-h)-1}] \quad (\text{A.6})$$

Because the left hand side is increasing in  $k$ ,<sup>32</sup> and the right hand side is decreasing in both  $h$  and  $k$ ,<sup>33</sup> we can conclude that equation (A.6) describes a negative-sloped locus in the plane  $\{k, h\}$ . We call this locus SS in Figure A.1. So far we have just replicated Azariadis-Drazen's claim for existence and uniqueness of an equilibrium in this economy. The equilibrium is given by the pair of capital intensity and training time at which SS and AA intersect in Figure A.1. Nevertheless, by including a positive population growth rate we can study how the equilibrium is perturbed by population growth. To obtain a negative relation between population and income per capita growth, we need the locus SS to move downward as  $n$  increases. That would be the case if  $s$  decreases as  $n$  increases in (A.6). The impact of  $n$  on  $s$  is not obvious. The population growth rate reduces both the second and third term in the right hand side of (A.6). The impact on the second term goes in the right direction

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<sup>32</sup>When the capital-labor elasticity of substitution is greater than the capital income share on output. See Azariadis and Drazen (forth.).

<sup>33</sup>By simple differentiation of second and third term with respect to  $h$  and recalling the properties of  $s$  stated above we can see that  $s$  is decreasing in  $h$ .

(reduction in second term reduces  $s$ ), but the impact on the third term does not (reduction in third term increases the saving function). The exact condition for an inverse relation between population and income per capita growth is

$$s_2 \frac{1-h}{[g(h)]^2} > -s_3 \quad (\text{A.7})$$

where  $s_i$  means partial derivative. For instance, for the CRRA and time separable class of utility function, and after some manipulation of the expression above (using A.3), the condition becomes

$$\left( \frac{g'(h)}{1+\delta} \right)^\gamma \frac{1-h}{[g(h)]^2} > 1 \quad (\text{A.8})$$

where  $\delta$  is the time preference parameter and  $\gamma$  the intertemporal elasticity of substitution (inverse of the relative risk aversion parameter). The condition will be satisfied if the function  $g$  is sufficiently concave, there is sufficiently high intertemporal substitutability and the time preference parameter is sufficiently low.



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